امتحان
التكامل بالتعويض

1. $\int 6x^2(2x^3-3)^4\ dx$:

a) $(2x^3-3)^5+C$
b) $\frac{1}{5}(2x^3-3)^5+C$
c) $5(2x^3-3)^5+C$
d) $-\frac{1}{5}(2x^3-3)^5+C$

2. $\int x\,e^{x^2+1}\ dx$:

a) $\frac{1}{2}e^{x^2+1}+C$
b) $2e^{x^2+1}+C$
c) $\frac{1}{2}e^{x^2+1}+C$
d) $-2e^{x^2+1}+C$

3. $\int \frac{1}{x\ln x}\ dx$:

a) $\ln|x|+C$
b) $\frac{1}{x}+C$
c) $2\ln|x|+C$
d) $\ln|\ln x|+C$

4. $\int \frac{x^4}{(x^5+9)^3}\ dx$:

a) $\frac{-1}{20(x^5+9)^2}+C$
b) $\frac{1}{20}(x^5+9)^4+C$
c) $\frac{-1}{10(x^5+9)^2}+C$
d) $\frac{-1}{20}(x^5+9)^4+C$

5. $\int \cos^4 x\sin x\ dx$:

a) $\frac{-\cos^5 x}{5}+C$
b) $\frac{-\sin^5 x}{5}+C$
c) $\frac{\cos^5 x}{5}+C$
d) $\frac{\sin^5 x}{5}+C$

6. $\int \frac{3x-3}{\sqrt{x^2-2x+4}}\ dx$:

a) $\frac{3}{\sqrt{x^2-2x+4}}+C$
b) $3\sqrt{x^2-2x+4}+C$
c) $\frac{-3}{\sqrt{x^2-2x+4}}+C$
d) $-3\sqrt{x^2-2x+4}+C$

7. $\int e^x(2+e^x)^5\ dx$:

a) $\frac{(2+e^x)^5}{5}+C$
b) $\frac{(2+e^x)^4}{4}+C$
c) $\frac{(2+e^x)^3}{3}+C$
d) $\frac{(2+e^x)^6}{6}+C$

8. $\int x^3\cos(x^4-5)\ dx$:

a) $-\frac{1}{4}\sin(x^4-5)+C$
b) $\frac{1}{4}\cos(x^4-5)$
c) $\frac{1}{4}\sin(x^4-5)+C$
d) $-\frac{1}{4}\cos(x^5-5)+C$

9. $\int_{-3}^{3} \frac{x}{\sqrt{x^2+9}}\ dx$:

$a)\ 0$
$b)\ 2$
$c)\ -2$
$d)\ 4$

10. إذا كان: $\int_{0}^{k} 3x^2 e^{x^3}\ dx = e^{64} - 1$، فإن قيمة k :

$a)\ 64$
$b)\ 8$
$c)\ 2$
$d)\ 4$

التكامل بالتعويض

1] $\int 6x^2 (2x^3-3)^4 \, dx$

الفرض
$u = 2x^3 - 3$
$\frac{du}{dx} = 6x^2$
$\frac{du}{6x^2} = dx$

$\int \cancel{6x^2} (u)^4 \frac{du}{\cancel{6x^2}}$

$= \frac{1}{5} u^5 + c = \frac{1}{5} (2x^3-3)^5 + c$

$\boxed{b}$

2] $\int x e^{x^2+1} \, dx$

الفرض
$u = x^2+1$
$\frac{du}{dx} = 2x$
$\frac{du}{2x} = dx$

$\int \cancel{x} e^u \frac{du}{2\cancel{x}} = \int \frac{1}{2} e^u \, du$

$= \frac{1}{2} e^u + c = \frac{1}{2} e^{x^2+1} + c$

$\boxed{a}$ أو $\boxed{c}$ مكرر

3] $\int \frac{1}{x \ln x} \, dx$

الفرض
$u = \ln x$
$\frac{du}{dx} = \frac{1}{x}$
$x\,du = dx$

$\int \frac{1}{\cancel{x} u} \cdot \cancel{x} \, du = \int \frac{1}{u} \cdot du$

$= \ln |u| + c = \ln |\ln x| + c$

$\boxed{d}$

حل

4] $\int \frac{x^4}{(x^5+9)^3}\,dx = \int x^4 (x^5+9)^{-3}\,dx$

الفرضة

$u = x^5+9$

$\frac{du}{dx} = 5x^4$

$\frac{du}{5x^4} = dx$

$\int \cancel{x^4}\,(u)^{-3}\,\frac{du}{5\cancel{x^4}}$

$\int \frac{1}{5}(u)^{-3}\,du = \frac{1}{5}\,\frac{u^{-2}}{-2} + c$

$= \frac{-1}{10(u)^2} + c = \frac{-1}{10(x^5+9)^2} + c$

$\boxed{c}$

5] $\int (\cos x)^4 \sin x\,dx$

الفرضة

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{du}{-\sin x} = dx$

$\int (u)^4\,\cancel{\sin x}\,\frac{du}{-\cancel{\sin x}}$

$\int -(u)^4\,du$

$-\frac{1}{5}u^5 + c$

$= \frac{-1}{5}(\cos x)^5 + c = \frac{-1}{5}\cos^5 x + c$

$\boxed{a}$

6] $\int (3x-3)(x^2-2x+4)^{-1/2}\,dx$

الفرض
$u = x^2-2x+4$
$\frac{du}{dx} = 2x-2$
$\frac{du}{2x-2} = dx$

$\int (3x-3)(u)^{-1/2}\frac{du}{2x-2}$

$\int 3\cancel{(x-1)}\,u^{-1/2}\cdot\frac{du}{2\cancel{(x-1)}}$

$\int \frac{3}{2}u^{-1/2}\,du$

$\frac{3}{2}\cdot\frac{2}{1}u^{1/2}+c = 3\sqrt{u}+c = 3\sqrt{x^2-2x+4}+c$ $\boxed{b}$

7] $\int e^x(2+e^x)^5\,dx$

الفرض
$u = 2+e^x$
$\frac{du}{dx} = e^x$
$\frac{du}{e^x} = dx$

$\int \cancel{e^x}(u)^5\frac{du}{\cancel{e^x}}$

$\frac{1}{6}u^6+c$

$\frac{1}{6}(2+e^x)^6+C$ $\boxed{d}$

30

8] $\int x^3 \cos(x^4-5)\,dx$

الفرض

$u = x^4-5$

$\frac{du}{dx} = 4x^3$

$\frac{du}{4x^3} = dx$

$\int \cancel{x^3} \cos(u) \frac{du}{4\cancel{x^3}}$

$\int \frac{1}{4} \cos(u)\,du$

$\frac{1}{4} \sin u + c \quad = \frac{1}{4} \sin(x^4-5)+c$

9] $\int_{-3}^{3} \frac{x}{(x^2+9)^{1/2}}\,dx = \int_{-3}^{3} x(x^2+9)^{-1/2}\,dx$

الفرض

$u = x^2+9$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int_{-3}^{3} \cancel{x}(u)^{-1/2} \frac{du}{2\cancel{x}}$

$\int_{-3}^{3} \frac{1}{2}(u)^{-1/2}du = \frac{1}{2}\cdot\frac{2}{1}u^{\frac{1}{2}}\Big]_{-3}^{3}$

$\sqrt{x^2+9}\Big]_{-3}^{3}$

$(\sqrt{18}) - (\sqrt{18})$

$= zero$

4

10] $\int_0^K 3x^2 e^{x^3} dx = e^{64} - 1$

الفرض

$u = x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$\int_0^K \cancel{3x^2} e^u \frac{du}{\cancel{3x^2}}$

$\int_0^K e^u du$

$e^u \Big]_0^K = e^{x^3} \Big]_0^K$

$(e^{K^3}) - (e^0) = e^{64} - 1$

$e^{\boxed{K^3}} - 1 = e^{\boxed{64}} - 1$

$\sqrt[3]{K^3} = \sqrt[3]{64}$

$\boxed{K = 4}$ $\boxed{d}$

Good Luck

Finito